NPTEL-Bangabasi College Local Chapter

WAVES & OPTICS ASSIGNMENT

1. Which of the following describes position of a point mass oscillating along X-axis?

a) $x(t) = A \cos\omega t + B \sin\omega t + C$

b) $x(t) = A \sinh(\omega t+\phi)$

c) $x(t) = R \exp(\omega t)$

d) None of the above

2. Which of the following is true regarding motion of a point mass oscillating along X-axis?

a) Net force on it is proportional to displacement from equilibrium position

b) Net force on it is proportional to negative of displacement from equilibrium position

c) Energy of the point mass is proportional to amplitude of the oscillation

d) None of the above

3. A block of mass 40kg is hanging vertically from one end of a massless spring of force constant 10N/m. The other end of the spring is attached to the ceiling. What is the time period of oscillation in seconds?

a) π

b) 2π

c) 3π

d) 4π

4. A point mass is oscillating due to a restoring force has amplitude 0.1m and period 0.8s. At t=0, the point mass is to the left of the equilibrium position and moving away from it. What is the position and direction of motion of the point mass at t=2s with respect to the equilibrium position?

a) Right, moving away

b) Right, moving towards

c) Left, moving towards

d) Left, moving away

5. A point mass is oscillating with amplitude 0.2m due to a restoring force of constant 100N/m. What is the total energy of the system?

a) 0.2J

b) 2J

c) 0.1J

d) 1J

6. A point mass of 2kg hangs vertically without oscillating from one end of a massless spring of force constant 18N/m. The other end is attached to the ceiling. The mass is pulled vertically from its equilibrium position (y=0) downwards from the ceiling to a position y=2 and then released. How much time does it take (in seconds) for the point mass to travel to the point y=1 after it is released?

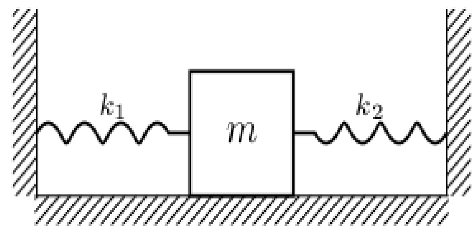
a) 2π

b) $2\pi/3$

c) $\pi/3$

d) $\pi/9$

7. A mass m resting on a frictionless horizontal surface is connected to rigid walls on either side by two springs of constants k_1 & k_2 as shown. The period of oscillation is



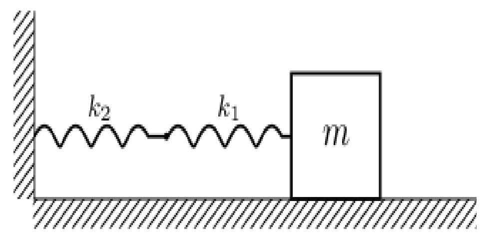
a) $2\pi\sqrt{m/(k_1+k_2)}$

b) $2\pi\sqrt{mk_1/(k_1k_2)}$

c) $2\pi\sqrt{m(k_1+k_2)/(k_1k_2)}$

d) $2\pi\sqrt{mk_2/(k_1k_2)}$

8. A mass m resting on a frictionless horizontal surface is connected to a rigid wall on the left side by two springs of constants k_1 & k_2 as shown. The period of oscillation is



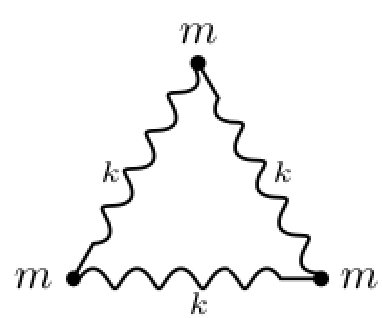
a) $2\pi\sqrt{m/(k_1+k_2)}$

b) $2\pi\sqrt{mk_1/(k_1k_2)}$

c) $2\pi\sqrt{m(k_1+k_2)/(k_1k_2)}$

d) $2\pi\sqrt{mk_2/(k_1k_2)}$

9. Three point masses m lie at rest on the vertices of an equilateral triangle on frictionless horizontal surface, connected by three springs of constant k as shown in figure. If each mass is simultaneously displaced by a small amount towards the centroid, what is the time period of small oscillations?



a) $2\pi\sqrt{m/k}$

b) $2\pi\sqrt{m/2k}$

c) $2\pi\sqrt{m/3k}$

d) $2\pi\sqrt{m/4k}$

10. Consider two oscillations of the form $x_1 = 3\sin(20\pi t + \pi/6), x_2 = 4\sin(20\pi t + \pi/2)$. The amplitude and phase of the superposition are respectively
a) √14, arctan(-3√3)
b) 1, arctan (-√3)
c) √37, arctan (-3√3/11)
d) None of the above

11. Which of the following is true regarding a viscous dissipative force that cause damping to an oscillator?
a) It is proportional to the velocity of the oscillator
b) It is proportional to the negative of the velocity of the oscillator
c) It is proportional to the displacement of the oscillator
d) It is proportional to the negative of the displacement of the oscillator

12. Which of the following is true for a damped oscillator?
a) Critically damped oscillator can exhibit oscillatory behavior
b) Critically damped oscillator is fastest to return to a position of zero displacement from the position of maximum displacement
c) Heavily damped oscillator is fastest to return to a position of zero displacement from the position of maximum displacement
d) Low damped oscillator is fastest to return to a position of zero displacement from the position of maximum displacement

13. In the equation $m\ddot{x} + \gamma\dot{x} + kx = 0$, what is the dimension of γ?
a) LT^{-1}
b) LT^{-2}
c) MLT^{-2}
d) MT^{-1}

14. In the equation $m\ddot{x} + \gamma\dot{x} + kx = 0$, what is the Q factor if m=25, γ=0.01, k=1?
a) 50
b) 1/50
c) 500
d) 1/500

15. Let the equation $m\ddot{x} + \gamma\dot{x} + kx = 0$ represent a lightly damped oscillator. What is the time after which amplitude reduces to half its value?
a) 2m (ln2)/γ

b) m (ln2)/γ
c) m (ln2)/2γ
d) 2γ (ln2)/m

16. A point mass of 2 units oscillates under a force of constant 50 units. By what % does the frequency decrease when a dissipative force of coefficient 12 units is also present?
a) 20%
b) 10%
c) 5%
d) 1%

17. The amplitude of a lightly damped oscillator decreases by 75% after four complete oscillations. What is the Q factor of the oscillator?
a) 4π ln4
b) 4 ln4
c) 4π/(ln4)
d) 4/(ln4)

18. The fractional change in frequency of an undamped oscillator when a light damping force acts on it has the value 2×10^{-6}. What is the Q-value of the oscillator?
a) 100
b) 125
c) 250
d) 500

19. Consider a closed ideal electrical circuit consisting of an inductor, capacitor & resistor. The Q value is
a) L / $\sqrt{(LCR)}$
b) R / $\sqrt{(LCR)}$
c) R / L$\sqrt{(CR)}$
d) L / R$\sqrt{(LC)}$

20. Which of the following would be the valid solution of the equation of motion $m\ddot{x} + \gamma\dot{x} + kx = 0$ when γ=10 and k = 5 units?
a) $5te^{-t}$
b) $5(1+t)e^{-t}$
c) te^{-2t}
d) $(1+t)\, e^{-t}$

21. Which of the following pairs of functions are out of phase?

a) cos(ωt) & cos(2ωt)
b) cos(ωt) & sin(ωt + π/2)
c) cos(ωt) & 2cos(ωt)
d) None of these

22. Which of the following is true regarding a forced oscillator?
a) Frequency of the external force at amplitude resonance is less than that at velocity resonance.
b) Frequency of the external force at amplitude resonance is greater than that at velocity resonance.
c) Frequency of the external force at amplitude resonance is equal to that at velocity resonance.
d) None of these

23. Let $\ddot{y} + \dot{y} + 4y = 5\cos\omega t$ be the equation of motion of a forced oscillator. For what value of ω will the velocity and the external force will be in phase?
a) 5
b) 4
c) 3
d) 2

24. Consider a closed ideal electrical circuit consisting of a series LCR circuit with R = 10Ω and an applied AC voltage V(t) = 8 cos(4t) volts. What is the maximum value of the average power input of the circuit?
a) 6.2W
b) 3.2W
c) 0.8W
d) 0.4W

25. Let $\ddot{y} + 4y = \cos 1.9t$ be the equation of motion of a undamped forced oscillator. What is frequency of oscillation in the steady state provided that y(0)=0 and v(0)=0?
a) 0.05
b) 1.9
c) 1.95
d) 2

26. The average power input of a forced oscillator is maximum if the frequency of the external force is ω_0 and is half the maximum value when frequency of the external force being 0.98 ω_0 1.02 ω_0. The Q-factor is
a) 20
b) 25

c) 40
d) 50

27. The Q-factor of a forced oscillator is 25 and its relaxation time is 50s. What is the natural frequency?
a) 4 Hz
b) 2 Hz
c) 1 Hz
d) 0.5 Hz

28. A point mass is oscillating due to a restoring force whose constant is 50N/m and an external force whose amplitude 10N. A dissipative force is also present. What is the maximum value of the displacement of the point mass from its equilibrium position during amplitude resonance if the Q-factor of the oscillator is 100?
a) 20
b) 0.2
c) 25
d) 0.25

29. Let $\ddot{y} + 4y = \cos 2t$ be the equation of motion of an undamped and forced harmonic oscillator. Which one of the following is a solution, given that y(0) = 0 and v(0) = 0?
a) y(t) = sin(2t) / 4
b) y(t) = cos(2t) /4
c) y(t) = t sin(2t) /4
d) y(t) = t cos(2t) /4

30. Let $m\ddot{x} + kx = F_0 \sin^2 \omega t$ be the equation of an undamped and forced oscillator with x(0) = 0 and v(0) = 0. Which one is the value of k at amplitude resonance?
a) ω_0^2
b) $2\omega_0^2$
c) $4\omega_0^2$
d) $8\omega_0^2$

31. Waves are propagating through a taut string of tension T & linear mass density μ. Dimension of T/μ is
a) LT^{-1}
b) L^2T^{-1}
c) LT^2
d) L^2T^{-2}

32. Which of the following is a solution to the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$?

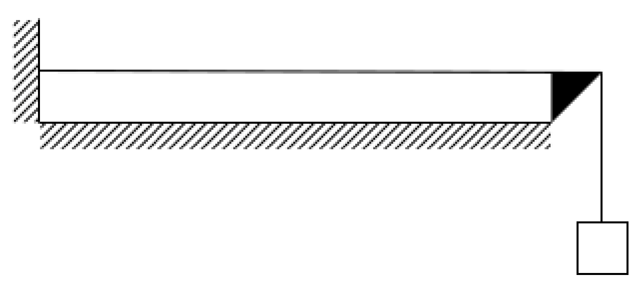
a) y = B sin(cx-t)
b) y = B sin (x-ct)
c) y = c sin (x-Bt)
d) None of above

33. Waves of velocity 80m/s and frequency 20 Hz are propagating through a taut string. What is the distance between the two points of the string which have phase difference π/6?
a) ¼ m
b) 1/3 m
c) ½ m
d) 2/3 m

34. A string of length 16 m is attached in a taut manner between two rigid walls. A standing wave is formed in the string of the form $y(x,t) = A\sin(4\pi ct/L)\sin(4\pi x/L)$, where A is amplitude and c is the speed of the wave. What is the greatest horizontal distance from either side of the wall where a node has been formed?
a) 4m
b) 8m
c) 12m
d) 14m

35. In a stretched string of tension 10N and mass density 0.1 Kg/m, the speed of the wave is
a) 0.01 m/s
b) 0.1 m/s
c) 10 m/s
d) 100 m/s

36. A string of mass density 0.02 Kg/m has one end attached to a fixed wall & the other passing over a wedge and connected to a 10 Kg mass hanging downwards. The length of the string between wall & wedge is 2m and it is plucked at a point here. The frequency of fundamental mode is



a) 70 Hz
b) 52.5 Hz
c) 35 Hz
d) 17.5 Hz

37. A message is represented by a wave packet. What is the speed with which the message is transmitted?
a) The smallest phase velocity of the wave packet
b) The largest phase velocity of the wave packet
c) The group velocity of the wave packet
d) None of these

38. The frequency range of a wave packet is 50 Hz. What is the maximum time interval for which the form of the wave packet is unaltered?
a) 1/25 s
b) 1/50 s
c) 2π/25 s
d) 4π/25 s

39. Waves are propagating through a taut string of length 2m, having mass density μ(x) = kx (where k is a constant and x is a distance from one end), tension 10N and total mass of 0.1 Kg. How much time does it take for a wave pulse to travel from one end of the string to the other?
a) 1/30s
b) 1/15s
c) 2/15s
d) 4/15 s

40. Consider a taut string of length L and mass per unit length μ. A standing wave in the string is represented by $y(x,t) = A\sin(4\pi ct/L)\sin(4\pi x/L)$, where A is the amplitude constant & c is the wave velocity. What is the total energy of a segment of the string between the two consecutive nodes?
a) $4\pi^2c^2A^2\mu^2/L$
b) $2\pi^2c^2A^2\mu^2/L$
c) $\pi^2c^2A^2\mu^2/L$
d) $4\pi^2c^2A^2\mu^2/2L$

Problems on Optics

1. An electromagnetic plane wave with λ = 1 mm is normally incident on a screen with two slits with spacing d = 3 mm.
a. How many maxima will be seen, at what angles to the normal?
b. Also solve the same problem by considering the situation where the wave is incident at 30◦ to the normal.

2. Two radio antennas separated by a distance d = 10 m emit the same signal at frequency ν with phase difference φ. Determine the values of ν and φ so that the radiation intensity is maximum in one direction along the line joining the two antennas while it is minimum along exactly the opposite direction. How do the maxima and minima shift of φ is reduced to half the earlier value?

3. A lens of diameter 5.0 cm and focal length 20 cm is cut into two identical halves. A layer 1 mm in thickness is cut from each half and the two lenses are joined again. The lens is illuminated by a point source located at the focus and a fringe pattern is observed on a screen 50 cm away. What is the fringe spacing and the maximum number of fringes that will be observed?

4. Two coherent monochromatic point sources are separated by a small distance, find the shape of the fringes observed on the screen when,
a) the screen is at one side of the sources and normal to the screen is along the line joining the two sources and b) when the normal to the screen is perpendicular to the line joining the sources.

5. The radiation from two very distant sources A and B is measured by the two antennas 1 and 2 separated by a distance'd'. The antennas operate at a wavelength λ. The antennas produce voltage outputs V_1 and V_2 which have the same phase and amplitude as the electric field E_1 and E_2 incident on the respective antennas. The voltages from the two antennas are combined $V= V_1 + V_2$ and applied to a resistance. The average power P dissipated across the resistance is measured. In this problem you can assume that the angle $2\theta \ll 1$ (in radians), which the sources subtend at the midpoint of the line joining the two antennas.
a. What is the minimum value of d (separation between the two antennas) at which P = 0?
b. Consider a situation when an extra phase φ is introduced in V_1 before the signals are combined. For what value of φ is P independent of d?

6. Find the condition for a dark fringe at any arbitrary P on the screen placed perpendicularly at a distance D from a source in Lloyds mirror arrangement. Also find the number of fringes observed on the screen. Assume source wavelength to be λ, and that it is at a distance r from the mirror edge at a tilt angle θ.

7. Starting from a central dark fringe, the eighth successive bright and dark fringes are observed at the center when one of the mirrors of a Michelson interferometer is moved 2.2 μm. Determine the wavelength of the light which is being used. (Ans. 5.5 Angstroms)

8. A Sodium lamp emits light at two neighboring wavelengths 5890A and 5896A. A Michelson interferometer is adjusted so that the fringes are in concordance. One of the mirrors is moved a distance Δd so that the fringes become discordant and concordant again. For what displacement

Δd are the fringes most discordant i.e. the fringe pattern becomes the faintest, and for what Δd does it become concordant again?

9. A Michelson interferometer illuminated by sodium light is adjusted so that the fringes are concordant with a central dark fringe. What is the angular radius of the first dark fringe if the order of the central fringe is m = 100 and m = 1000? What happens if a Michelson interferometer is illuminated by white light? Also consider the situation where d = 0 i.e. the two arms have the same length.

10. Newton's rings are observed by placing a plano-convex lens of radius of curvature 60 cm on top of a glass plate (both are having the same refractive index) with Sodium light (λ = 600 nm). Find the radius of the fourth bright ring. (Ans. 1.2 mm)

11. If the air film between the lens and the glass plate in the above problem is filled with a transparent liquid of refractive index 1.28, what will be the radius of the fifth bright ring? (Ans. 1.125 mm)

12. If the air film in the problem same is filled with Benzene and the sixth dark ring has radius 1.2 mm, find out refractive index of Benzene. What will be radius of the fourteenth bright ring if wavelength of the light is changed to λ = 504.2 nm? (Ans. 1.5, 1.65mm)

13. In the Newton's ring problem, suppose the refractive index of the material of the plano-convex lens is 1.48 and that of the lower glass plate is 1.52. What difference one would observe in the Newton's rings in this case compared to that when both have same refractive index of 1.50?

14. A thin coat of a transparent material of refractive index 1.4 is applied on one surface of a plane glass plate (refractive index 1.5) to reduce the reflection of orange light, λ = 616 nm. What should be the minimum thickness of this coat if the light rays are incident normally onto the glass plate and a coat of thickness less than 0.5 μm is almost impossible to apply. (Ans. 0.55μm)

15. Consider a situation where Young's double slit experiment is performed using light of wavelength 550 nm and d = 1 m. Calculate the visibility assuming a source of angular width 1′ and 1◦ . Plot I(θ) for both these cases.

16. A small aperture of diameter 0.1 mm at a distance of 1m is used to illuminate two slits with light of wavelength λ = 550 nm. The slit separation is d = 1 mm. What is the fringe spacing and the expected visibility of the fringe pattern? (5.5×10^{-4} rad, V = 0.95)

17. A source of unknown angular extent α emitting light at $\lambda = 550$ nm is used in a Young's double slit experiment where the slit spacing d can be varied. The visibility is measured for different values of d. It is found that the fringes vanish (V = 0) for d = 10 cm. What is the angular extent of the source? (Ans. 5.5×10^{-6} rad)

18. Estimate the coherence time τ_c and coherence length l_c for the following sources.

Source	λ (nm)	$\Delta\lambda$ (nm)
White light	550	300
Mercury arc	546.1	1.0
Argon ion gas laser	488	0.06
Red Cadmium	643.847	0.0007
Solid state laser	785	10^{-6}
He-Ne laser	632.8	10^{-6}

19. Assume that Kr-86 discharge lamp has roughly the following intensity distribution at various wavelengths, λ (in nm), $I(\lambda) = \dfrac{36 I_0}{36 + (\lambda - 605.616)^2 \times 10^8}$. Estimate the coherence length of Kr-86 source. (Ans. 0.3m)

20. An ideal Young's double slit (i.e. identical slits with negligible slit width) is illuminated with a source having two wavelengths, $\lambda_1 = 418.6$ nm and $\lambda_2 = 421.4$ nm. The intensity at λ_1 is double of that at λ_2.
a) Compare the visibility of fringes near order m = 0 and near order m = 50 on the screen (Ans. 1:0.5)
b) At what order(s) on the screen visibility of the fringes is poorest and what is this minimum value of the visibility. (Ans. 75, 225 etc. and 1/3)

21. An ideal Young's double slit (separation d between the slits) is illuminated with two identical strong monochromatic point sources of wavelength λ. The sources are placed symmetrically and far away from the double slit. The angular separation of the sources from the mid-point of the double slit is θ_s. Estimate θ_s so that the visibility of the fringes on the screen is zero. Can one have visibility almost 1 for a non zero θ_s.

22. For a slit of dimensions 1 mm × 1 cm, what are the positions of the first three diffraction minima's on either side of the central maxima? Use $\lambda = 550$ nm and 0.1 mm.

23. For a rectangular slit whose smaller dimension is D, what are the positions of the diffraction maxima for light of wavelength λ? (Ans. $\beta \sim \pm 1.43\pi, \pm 2.46\pi, \pm 3.47\pi$, etc.)

24. Calculate the ratio of intensities of the first intensity maximum and central maximum for the previous problem. (Ans. ~21)

25. Compare the angular resolutions of two circular apertures (D_1 = 1 mm and λ_1 = 550 nm) and (D_2 = 45 m and λ_2 = 1 m).

26. A plane wave of light with wavelength λ = 0.5 μm falls on a slit of width = 10 μm at an angle 30◦ to the normal. Find the angular position, with respect to the normal, of the first minima on both sides of the central maxima.

27. Plot the intensity profile as a function of θ for a double slit with d = 0.1mm and b = 0.025mm. Assume wavelength of the incident monochromatic light to be equal to 500nm. Keep $|\theta| < 2.5$ degrees. Locate the missing orders in this pattern.

28. Obtain the double slit pattern in the limit $b \rightarrow D$ and justify your answer intuitively

29. The collimator of a spectrometer has a diameter of 2cm. What would be the largest grating element for a grating, which would just resolve the Sodium doublet at the second order, using this spectrometer? (Sodium doublet: D_1 = 589.0nm and D_2 = 589.6nm, (Ans. d ~0.04mm)

30. Obtain the expression of intensity for a double slit with separation d between the slits and individual slit width D, as a special case of N=2.