Assignment 5

1. The value of the integral $\int_{\phi'=0}^{2\pi}\int_{\theta'=0}^{\pi}\frac{\cos\theta'\sin\theta' d\theta' d\phi'}{\sqrt{(R^2+r^2-2Rr(\cos\theta\cos\theta'+\sin\theta\sin\theta'\cos(\phi-\phi'))}}$ is: (assume $\phi-$symmetry)
   (a) $\dfrac{-R\cos\theta}{3r^2}$ for $r > R$, $\dfrac{-r\cos\theta}{3}$ for $r < R$
   (b) $\dfrac{R\cos\theta}{3r^2}$ for $r > R$, $\dfrac{r\cos\theta}{3}$ for $r < R$
   (c) $\dfrac{-2R\cos\theta}{3r^2}$ for $r > R$, $\dfrac{-2r\cos\theta}{3}$ for $r < R$
   (d) $\dfrac{2R\cos\theta}{3r^2}$ for $r > R$, $\dfrac{2r\cos\theta}{3R^2}$ for $r < R$.

2. A uniformly charged thin spherical shell carrying charge density σ is rotating at angular speed ω about the $z-$axis. Current density for the system is given by: (Spherical coordinates are used)
   (a) $\vec{j}(\vec{r}) = \sigma\omega R\sin\theta\hat{\phi}$
   (b) $\vec{j}(\vec{r}) = \sigma\omega R\cos\theta\hat{\phi}$
   (c) $\vec{j}(\vec{r}) = \sigma\omega r\delta(r-R)\sin\theta\hat{\phi}$
   (d) $\vec{j}(\vec{r}) = \sigma\omega r\delta(r-R)\cos\theta\hat{\phi}$.

3. The value of the integral $\int\limits_0^{2\pi}\int\limits_{-\infty}^{\infty}\frac{d\phi' dz'}{\sqrt{r^2+R^2-2Rr\cos(\phi-\phi')+(z-z')^2}}$ is: (Hint: Use Gauss's law for a uniformly charged infinitely long cylindrical shell)
   (a) $2\pi\ln(\frac{R}{r})$ for $r > R$, 0 for $r \leq R$
   (b) $2\ln(\frac{R}{r})$ for $r > R$, 0 for $r \leq R$
   (c) $\pi\ln(\frac{R}{r})$ for $r > R$, $\pi(R-r)$ for $r < R$
   (d) $2\ln(\frac{R}{r})$ for all r.

4. A thin disc carries uniform magnetization $\vec{M}$ perpendicular to its flat surface. If it is kept in the $xy-$plane with its axis on the $z-$axis with $\vec{M} = M\hat{z}$, the auxiliary field $\vec{H}$ and magnetic field $\vec{B}$ just outside the upper surface on the axis is:
   (a) $\vec{H} = \vec{M}$, $\vec{B} = 0$
   (b) $\vec{H} = \vec{M}$, $\vec{B} = \mu_0\vec{M}$
   (c) $\vec{H} = 0$, $\vec{B} = 0$
   (d) $\vec{H} = -\vec{M}$, $\vec{B} = 0$.

5. For the disc given in question (4), the fields inside the disc near its axis are:

   (a) $\vec{H} = -\vec{M}$, $\vec{B} = 0$

   (b) $\vec{H} = \vec{M}$, $\vec{B} = \mu_0 \vec{M}$

   (c) $\vec{H} = \vec{M}$, $\vec{B} = 2\mu_0 \vec{M}$

   (d) $\vec{H} = 0$, $\vec{B} = 0$.

6. Consider a long cylindrical rod of length L and radius R (where R << L). Take its axis to be the z axis. It carries magnetization $\vec{M} = M\hat{x}$. Using cylindrical co ordinates find out $\nabla.\vec{H}$ at s, far from the circular surface:

   (a) $M \sin\phi \delta(s - R)$

   (b) $-M \cos\phi \delta(s - R)$

   (c) $M \sin\phi \delta(s - R)$

   (d) $M \cos\phi \delta(s - R)$.

7. The auxiliary field $\vec{H}$ and magnetic field $\vec{B}$ inside the magnetized cylinder in Question no 6 is:

   (a) $\vec{H} = -\vec{M}/2$, $\vec{B} = \frac{\mu_0 \vec{M}}{2}$

   (b) $\vec{H} = -\vec{M}$, $\vec{B} = 0$

   (c) $\vec{H} = \vec{M}$, $\vec{B} = 2\mu_0 \vec{M}$

   (d) $\vec{H} = 0$, $\vec{B} = \mu_0 \vec{M}$.

8. A superconductor has zero magnetic field inside it. If a solid sphere of superconducting material is placed in a uniform magnetic field $\vec{B}$ then $\vec{M}$ and $\vec{H}$ developed in the sphere are:

   (a) $\vec{M} = 0$, $\vec{H} = 0$

   (b) $\vec{M} = -\vec{B}$, $\vec{H} = \vec{B}$

   (c) $\vec{M} = \frac{-2\vec{B}}{3}$, $\vec{H} = \frac{2\vec{B}}{3}$

   (d) $\vec{M} = \frac{-3\vec{B}}{2}$, $\vec{H} = \frac{3\vec{B}}{2}$.

9. Value of χ_M for a superconductor is:

   (a) 0

   (b) -1

   (c) μ_0

   (d) 1.

10. If a magnetic dipole m$\hat{z}$ is kept at the origin in a magnetic material of susceptibility χ_M, the auxiliary field $\vec{H}$ at $r \neq 0$ satisfies:

(a) $\nabla.\vec{H} = 0$ and $\nabla \times \vec{H} = 0$

(b) $\nabla.\vec{H} = 0$ and $\nabla \times \vec{H} \neq 0$

(c) $\nabla.\vec{H} \neq 0$ and $\nabla \times \vec{H} \neq 0$

(d) $\nabla.\vec{H} \neq 0$ and $\nabla \times \vec{H} = 0$.

11. The auxiliary field $\vec{H}$ for point dipole in problem (10) is:

(a) $\vec{H} = 0$

(b) $\vec{H} = \chi_M \frac{3(\vec{m}.\hat{r})\hat{r} - \vec{m}}{4\pi r^3}$

(c) $\vec{H} = \frac{3(\vec{m}.\hat{r})\hat{r} - \vec{m}}{4\pi r^3}$

(d) $\vec{H} = \frac{\mu_0}{4\pi} \chi_M \frac{3(\vec{m}.\hat{r})\hat{r} - \vec{m}}{r^3}$.