Assignment 4

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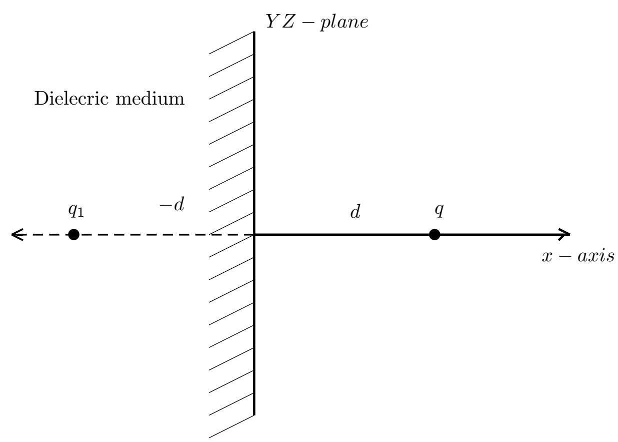
Q1. A point charge q is in front of a dielectric semi-infinite slab of dielectric constant K. The distance between the plane and the charge is d. Take the plane to be in the $YZ-$plane and the charge on the $x-$axis. To calculate the potential and electric field in $x \geq 0$ region, we place an image charge q_1 at $x = -d$ and to calculate the potential and the field in $x \leq 0$ region, we replace q by q_2. To satisfy the required boundary conditions we need:



(a) $q_1 = -q\,,\ q_2 = q$

(b) $q_1 = -\left(\frac{K-1}{K+1}\right) q\,,\ q_2 = \left(\frac{2K}{K+1}\right) q$

(c) $q_1 = \left(\frac{K-1}{K+1}\right) q\,,\ q_2 = -\left(\frac{2K}{K+1}\right) q$

(d) $q_1 = -\left(\frac{K-1}{K+1}\right) q\,,\ q_2 = \left(\frac{2}{K+1}\right) q.$

Q2. A solid infinitely long cylinder with its axis on the $z-$axis carries a polarization $\vec{P} = P_0\hat{x}$. The displacement vector inside the dielectric is:

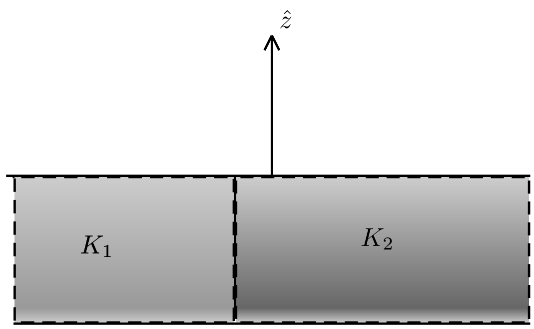
(a) $P_0\hat{x}$

(b) $\frac{1}{2}P_0\hat{x}$

(c) $-\frac{1}{2}P_0\hat{x}$

(d) $\frac{3}{2}P_0\hat{x}$.

Q3. A parallel plate capacitor has plates perpendicular to the $z-$axis and two dielectric slabs of dielectric constants K_1 and K_2 are placed in between (see figure). Ignoring fringing effects the boundary condition at the dielectric interface is:



(a) $E_{1z} = E_{2z}$

(b) $D_{1x} = D_{2x} \neq 0$

(c) $D_{1y} = D_{2y}$

(d) $D_{1z} = D_{2z}$.

Q4. A sphere carries a polarization $\vec{P} = P_0\hat{z}$. Electric field just outside its equator in the plane perpendicular to $\hat{z}$ is:

(a) $\frac{P_0}{\epsilon_0}\hat{z}$

(b) $-\frac{P_0}{2\epsilon_0}\hat{z}$

(c) $-\frac{P_0}{3\epsilon_0}\hat{z}$

(d) $-\frac{P_0}{\epsilon_0}\hat{z}$.

Q5. If a point charge Q is kept at the centre of a dielectric sphere with radius R (dielectric constant K) the potential $V(r)$, where r is measured from the centre of the sphere, is: (Take $V(\infty) = 0$)

(a) $V(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{Kr}$ for all r

(b) $V(r) = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{Q}{Kr} & \text{if } r > R \\ \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{1-K}{KR}\right] & \text{if } r \leq R \end{cases}$

(c) $V(r) = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{Q}{r} & \text{if } r > R \\ \frac{1}{4\pi\epsilon_0}\frac{Q}{Kr} & \text{if } r \leq R \end{cases}$

(d) $V(r) = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{Q}{r} & \text{if } r > R \\ \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{Kr} - \frac{K-1}{KR}\right] & \text{if } r \leq R \end{cases}$.

Q6. Consider a long cylinder with its axis on the $z-$axis. It is made of a dielectric material of electric susceptibility χ_e. If it is put in a uniform electric field $\vec{E} = E_0\hat{y}$, the electric field $\vec{E}$, polarization $\vec{P}$ and displacement current $\vec{D}$ are, respectively,

(a) $E\hat{y}\,,\ \chi_e\epsilon_0 E\hat{y}\,,\ (1+\chi_e)\epsilon_0\vec{E}$

(b) $\frac{E}{1+\chi_e}\hat{y}\,,\ \frac{\chi_e\epsilon_0}{1+\chi_e}\vec{P}\,,\ \epsilon_0\vec{E}$

(c) $\frac{2}{2+\chi_e}E\hat{y}\,,\ \frac{2\chi_e}{2+\chi_e}\epsilon_0 E\hat{y}\,,\ 2\left(\frac{1+\chi_e}{2+\chi_e}\right)\epsilon_0 E\hat{y}$

(d) $\frac{E}{2+\chi_e}\hat{y}\,,\ \frac{\chi_e\epsilon_0}{2+\chi_e}E\hat{y}\,,\ \left(\frac{1+\chi_e}{2+\chi_e}\right)E\hat{y}$.

Q7. Consider the field inside and outside a long solenoid carrying a surface current K. The magnetic field inside the solenoid is $\mu_0 K\hat{z}$ and the axis of the solenoid coincides with the $z-$axis. If R is the radius of the solenoid, then on the basis of these facts and Biot-Savart law the value of the integral

$$\int_{-\infty}^{\infty} \mathrm{d}z' \int_0^{2\pi} \mathrm{d}\phi' \frac{R - r\cos(\phi-\phi')}{(z^2 + R^2 + r^2 - 2rR\cos(\phi-\phi'))^{3/2}}$$

is:

(a) $\frac{4\pi}{R}$ for all r

(b) $\frac{4\pi}{R}$ for $r < R$ and 0 for $r > R$

(c) $\frac{2\pi}{R}$ for $r < R$ and 0 for $r > R$

(d) $\frac{1}{R}$ for $r < R$ and 0 for $r > R$.

Q8. A disc with its centre at the origin and the axis as the $z-$axis carries a uniform charge density σ. If it is rotating with an angular velocity ω then the magnetic field on the $z-$axis at $z = h$ will be:

(a) $\frac{\mu_0 \sigma \omega}{2} \left[\frac{R^2+2h^2}{\sqrt{R^2+h^2}} - 2h \right]$

(b) $\mu_0 \sigma \omega \left[\sqrt{R^2+h^2} - h \right]$

(c) $2\mu_0 \sigma \omega \left[\sqrt{R^2+h^2} - h \right]$

(d) $\frac{\mu_0 \sigma \omega R^2}{\sqrt{R^2+h^2}}$.

Q9. Consider a thin solenoid of length L and radius a ($L >> a$) bent into the shape of a ring so that it makes a toroid which is kept with its centre at the origin and the axis on the $z-$axis with the magnetic field in it being in the $\hat{\phi}$ direction. The solenoid carries a current I and it has N turns. Taking into account the similarity of the equations $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ and $\vec{\nabla} \times \vec{A} = \vec{B}$, the vector potential for this toroid on the $z-$axis at z will be:

(a) $\frac{\mu_0 L a^2 N I}{2(L^2+z^2)^{3/2}}$

(b) $\frac{\mu_0 L a^2 N I}{(L^2+4\pi z^2)^{3/2}}$

(c) $\frac{\mu_0 \pi L a^2 N I}{4(L^2+4\pi z^2)^{3/2}}$

(d) $\frac{\mu_0 \pi L a^2 N I}{2(L^2+4\pi^2 z^2)^{3/2}}$.

Q10. Magnetic vector potential on the $z-$axis of the uniformly charged disc of Q8 is:(Use $\nabla^2 \vec{A} = -\mu_0 \vec{J}$ to calculate it.)

(a) $\frac{\mu_0 \sigma \omega z^2}{R} \hat{\phi}$

(b) 0

(c) $\frac{\mu_0 \sigma \omega R^2}{\sqrt{R^2+z^2}} \hat{z}$

(d) $\mu_0 \sigma \omega z \hat{z}$.