Assignment 2

Questions to be submitted

1. For a given field $\vec{F} = 2x\hat{\imath} + 3y\hat{\jmath}$ in a region of space, the dot product $\vec{F} \cdot \mathrm{d}\vec{s}$, where $\mathrm{d}\vec{s}$ represents an infinitesimal area element on a cylinder of radius R at an angle of ϕ, is:

   A. $R^2 \,\mathrm{d}\phi \,\mathrm{d}z$

   B. $2R^2 \cos^2\phi \,\mathrm{d}\phi \,\mathrm{d}z$

   C. $R^2(2 - \sin^2\phi) \,\mathrm{d}\phi \,\mathrm{d}z$

   D. $R^2(2 + \sin^2\phi) \,\mathrm{d}\phi \,\mathrm{d}z$

2. The flux of the field in problem 1 over a quarter of the cylindrical surface extending from $\phi = \frac{\pi}{4}$ to $\phi = \frac{3\pi}{4}$ is: (Take the length of the cylinder = 1)

   A. πR^2

   B. R^2

   C. $\frac{5\pi R^2}{4} + \frac{R^2}{2}$

   D. $\frac{5\pi R^2}{4}$

3. If the density of the matter distributed in a part of space is given as $Cx^2|z|$, where C is a constant, the mass of an infinitesimal volume element in spherical polar coordinates is given as:

   A. $Cr^3 \cos^2\theta \mid \cos\theta \mid \sin^2\phi \,\mathrm{d}\phi \,\mathrm{d}r$

   B. $Cr^5 \sin\theta \cos^3\theta \cos^3\phi \,\mathrm{d}\phi \,\mathrm{d}\theta \,\mathrm{d}r$

   C. $Cr^5 \sin\theta \cos^3\theta \,\mathrm{d}\phi \,\mathrm{d}\theta \,\mathrm{d}r$

   D. $Cr^5 \sin^3\theta \mid \cos\theta \mid \cos^2\phi \,\mathrm{d}\phi \,\mathrm{d}\theta \,\mathrm{d}r$

4. Mass of a sphere of radius R centred around the origin with density distribution $Cx^2|z|$ is:

   A. $\frac{\pi C R^6}{12}$

   B. $\frac{C R^6}{16}$

   C. $\frac{\pi C R^6}{6}$

   D. $\frac{\pi C R^6}{8}$

5. Surface integral of vector field $\vec{V}(x,y) = x^2\,\hat{\imath} + xyz\,\hat{\jmath} + y^2 z\,\hat{k}$ over a unit sphere centred at the origin is (Use divergence theorem and spherical polar coordinates for integration):

   A. $\frac{4\pi}{3}$

   B. $\frac{4\pi}{15}$

   C. $\frac{4\pi}{5}$

   D. $\frac{4\pi}{10}$

6. For a vector field $\vec{V}(x,y) = x^2 y\,\hat{\imath} + (x-y)\,\hat{\jmath} + c(x+y)\,\hat{k}$, where c is a constant. The line integral $\oint \vec{V} \cdot \vec{dl}$ going anticlockwise over a square of side a, with two of the sides on the x and y axes, is (Use Stoke's theorem):

   A. a^2

   B. $a^2(1 + ca^2)$

C. $a^2(1 - \frac{a^2}{3})$

D. $a^2(1 + \frac{ca^2}{3})$

7. A vector field is given as $\vec{F} = 2xyz\,\hat{i} + x^2z\,\hat{j} + x^2y\,\hat{k}$. The corresponding potential to this field $V(x, y, z)$ up to a constant is:

   A. $3x^2yz$

   B. $-x^2yz$

   C. $2x^2yz$

   D. $\frac{x^2yz}{2}$

8. For a given potential function $z - 2x^2 - 3y^2$, a constant potential surface passing through $x = 1, y = 2$ and $z = 1$ will be described by:

   A. $z - 2x^2 - 3y^2 = 0$

   B. $z - 2x^2 - 3y^2 = constant$

   C. $2x^2 + 3y^2 - z = 13$

   D. $z - 2x^2 - 3y^2 = 5$

9. Using the gradient operator, one finds that the unit vector normal to the curve defined by $4x^2 + 9y^2 = 36$ at $(\frac{3}{2},\ \sqrt{3})$ is:

   A. $\frac{2}{\sqrt{31}}\hat{i} + \frac{3\sqrt{3}}{\sqrt{31}}\hat{j}$

   B. $\sqrt[3]{\frac{3}{31}}\hat{i} + \frac{2}{\sqrt{3}}\hat{j}$

   C. $\sqrt[3]{\frac{3}{31}}\hat{i} - \frac{2}{\sqrt{3}}\hat{j}$

   D. $\frac{2}{\sqrt{3}} - \sqrt{\frac{3}{31}}\hat{j}$

10. The electrostatic potential on the axis of a uniformly charged disc with uniform charge density σ and radius R at a height z from its centre is:

    A. $-\frac{\sigma}{2\epsilon_o}|z|$

    B. $\frac{\sigma}{2\epsilon_o}\sqrt{R^2 + z^2}$

    C. $\frac{\sigma}{2\epsilon_o}[\sqrt{R^2 + z^2} - |z|]$

    D. $\frac{\sigma}{2\epsilon_o}R$

11. Using the electrostatic potential derived for a charged shell, the result for electrostatic potential $V(r)$ for a uniformly charged sphere of radius R is obtained, is given as

    A. $-\frac{1}{4\pi\epsilon_o}\frac{Q}{r}$ for all r

    B. $\frac{1}{4\pi\epsilon_o}\frac{Q}{R}$ for $r \leq R$ and $\frac{1}{4\pi\epsilon_o}\frac{Q}{r}$ for $r \geq R$

    C. $\frac{1}{4\pi\epsilon_o}[\frac{30}{R} - 2\frac{Qr^2}{R^2}]$ for $r \leq R$

    D. $\frac{1}{4\pi\epsilon_o}[\frac{3Q}{2R} - \frac{1}{2}\frac{Qr^2}{R^3}]$ for $r \leq R$ and $\frac{1}{4\pi\epsilon_o}\frac{Q}{r}$ for $r \geq R$